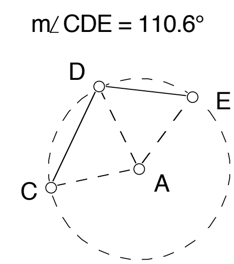
Exploration 4.3—Triangles Defined by Spokes

In the preceding exploration, you built a right triangle and then fit it into a circle. This exploration provides another point of view: You start with a circle and make some right triangles.

Investigation 1—Spokes and Right Triangles

You will make some spokes of equal length sticking out from point *A* and then study the situation when the angle formed by the endpoints of the spokes is 90 degrees. This will help explain the Carpenter's Construction.

Construction



- Draw circle *AB*. Hide point *B*.
- Place three points *C*, *D*, and *E* on the circumference of the circle and draw segments (radii) *AC*, *AD*, *AE*.
- Make the circle and radii dashed.

  This leaves three segments of equal length with a common endpoint *A*, which you can drag around.
- Draw segments *CD* and *DE* and measure angle *CDE*.

Experiment

- Move the points *C*, *D*, and *E* around and look for configurations in which angle *CDE* is approximately 90 degrees.

☞ What do you observe about the points *C*, *A*, and *E* when angle *CDE* is 90 degrees? Think about why this is true in the light of earlier explorations.

- Start with the figure in a position in which angle *CDE* is approximately 90 degrees. Leaving points *C* and *E* fixed, move the point *D*. What happens to the angle?

Conclusions

Q1. From your observations, write a conjecture describing the relationship among the points *C*, *A*, and *E* when angle *CDE* is 90 degrees.

Q2. How can you use the results of the previous paper-folding experiment to explain why your conjecture is true?

Q3. Use your conjecture to explain why the 90-degree angle *CDE* does not change when point *D* is moved and points C and *E* are stationary.

Explore More—Other Spoke Angles

In the preceding experiment you studied the figure *ACDE* when angle *CDE* is a right angle.

> *An angle CDE, where the points C, D, and E are on a circle, is called an* **inscribed angle in the circle**.

Continue the experiment with the same figure when the inscribed angle *CDE* is some angle other than a right angle. Here are some questions to investigate.

Q1. If you move point *D* and leave the others fixed, how does angle *CDE* change?

Q2. Label the equal angles in the triangles *ACD* and *ADE* and show how relations among the angles can help explain the behavior of angle *CDE* in Q1.

Q3. Conjecture how angle *CDE* is related to angle *CAE*. Explain why your conjecture is true using the relations in Q2.